SPONTANEOUS SCHERK-SCHWARZ SUPERSYMMETRY BREAKING AND RADION STABILIZATION *

MARIANO QUIROS

Theoretical Physics Group
ICREA-IFAE,
E-08193 Bellaterra (Barcelona) Spain
E-mail: quiros@ifae.es

In this talk I review the issues of supersymmetry breaking and radion stabilization in a five dimensional theory compactified on the $\mathbb{Z}_2$ orbifold. Supersymmetry breaking by Scherk-Schwarz boundary conditions is interpreted as spontaneous breaking of local supersymmetry by the Hosotani mechanism. The auxiliary field responsible for spontaneous supersymmetry breaking is inside the five-dimensional off-shell minimal supergravity multiplet. Different ways of fixing the supersymmetry breaking order parameter are analyzed. In the presence of supersymmetry breaking the one-loop effective potential for the radion has a minimum that fixes its vacuum expectation value. The radion is stabilized in a metastable Minkowski$_4$ minimum (versus the AdS$_4$ vacuum) with a mass in the meV range making it interesting for future deviations from the gravitational inverse-square law.

1. Introduction

Higher dimensional theories share the general problem of how to fix the radii of compact dimensions and, if they are supersymmetric, of how to break supersymmetry. While the first question is dependent on the second one, for in the absence of supersymmetry breaking the radion potential is flat and its vacuum expectation value (VEV) undetermined, the second question finds a very elegant solution in the Scherk-Schwarz (SS) mechanism [1] where supersymmetry is broken by global effects [2]. In this talk I will review possible solutions to both questions and what they imply for radion phenomenology. I will use as a prototype a five-dimensional (5D) theory

*Based on plenary talks presented at *SUSY 2003: Supersymmetry in the Desert*, held at the University of Arizona, Tucson, AZ, June 5-10, 2003; and at *IPPP Workshop on: String Phenomenology 2003*, held at the University of Durham, U.K., July 29-August 4, 2003. To appear in the Proceedings.

compactified on a $\mathbb{Z}_2$ orbifold with a flat geometry. This corresponds to 5D "ungauged" supergravity where the gravitino does not have any tree-level mass-term either in the bulk or localized at the boundaries.

2. Scherk-Schwarz supersymmetry breaking as spontaneous breaking

Scherk-Schwarz supersymmetry breaking of a 5D theory compactified on $S^1/\mathbb{Z}_2$ can be interpreted as a spontaneous breaking of 5D local supersymmetry [3]. To interpret SS supersymmetry breaking as a Hosotani mechanism [4] one has to go to the off-shell version of 5D N=1 supergravity where the $SU(2)_R$ automorphism of global supersymmetry is gauged by a tripet of auxiliary fields $\vec{V}_M$. The ungauged version of this theory has been recently formulated [5,6] and found that two multiplets are necessary: the minimal supergravity multiplet $(40_B + 40_F)$ and a tensor multiplet $(8_B + 8_F)$ with appropriate parities dictated by the orbifold group invariance of the theory. The only physical fields are in the gravitational multiplet: the graviton e_M^A, the gravitino ψ_M^i (where the index i transform as an $SU(2)_R$ doublet), and the graviphoton B_M. All other fields are auxiliary: in particular the relevant fields for supersymmetry breaking are the even fields $V_5^{1,2}$ that constitute the F-term of the radion superfield. SS supersymmetry breaking has also been studied in the context of gauged (AdS$_5$) supergravity [7,8].

In particular in the background of V_5^2 the Goldstino is provided by the fifth component of the gravitino (ψ_5) as it is obvious from the local supersymmetric transformation $\delta_\xi \psi_5 = \mathcal{D}_5 \xi + \cdots = i\sigma^2 V_5^2 \xi + \cdots$ A local supersymmetry transformation with parameter $\xi \equiv -(\mathcal{D}_5)^{-1}\psi_5$ gauges ψ_5 away and gives a mass to the gravitino. This defines the "super-unitary" gauge where ψ_5 has been "eaten" by the four-dimensional gravitino ψ_μ. In fact using the coupling of V_5^2 to the gravitino field through the covariant derivative $\mathcal{D}_5$ one obtains gravitino mass eigenvalues for the Kaluza-Klein modes where $m_{3/2}^{(0)} \propto \langle V_5^2 \rangle$ and $\langle V_5^2 \rangle$ can be identified with ω/R where R is the physical radius of the extra dimension and ω the SS parameter. I will next review different procedures for fixing both the SS parameter ω and the physical orbifold radius R.

3. Fixing the Scherk-Schwarz parameter

In Ref. [9] we described how to fix the SS parameter using 5D off-shell supergravity tools. There we explained how the tensor multiplet formalism and its dual, the linear multiplet one, in 5D supergravity are not equiv-

alent in the presence of a non-trivial cohomology, as that possessing the orbifold $S^1/\mathbb{Z}_2$. In particular using the tensor field B_{MNP} (tensor multiplet) and its field equation the one-form V_M is closed ($dV = 0$). Since the space is cohomologically non-trivial that form can be non-exact. In that case the VEV V_5^2 has a non-vanishing, but tree-level undetermined, Wilson flux as $\frac{1}{2\pi}\oint dx^5 \langle V_5^2 \rangle \equiv 2\omega$. In this case supersymmetry is broken but tree-level undetermined. Supersymmetry can then be broken by radiative corrections [10,11] as in the Hosotani breaking of a gauge theory. Of course this procedure does not violate any non-renormalization theorem since the tree-level potential is flat and for $\langle V_5^2 \rangle = 0$ the vacuum energy is zero.

In the linear multiplet formalism the tensor B_{MNP} is traded by the vector W_M. SS supersymmetry breaking is based on an intriguing property of the linear multiplet. I will first concentrate on a vector field E_M with vanishing field strength, $dE = 0$. This defines a Maxwell multiplet where all other components are equal to zero. This configuration is left invariant under local supersymmetry since local supersymmetric transformations only depend on E_M through its field strength, dE. This multiplet has no physical degrees of freedom but on non-simply connected spaces it can have a non-vanishing flux $\int E_M dx^M$. We call it "flux-multiplet". Under $\mathbb{Z}_2$ compactification with even E_5 the flux multiplet reduces to the constant multiplet that is known to be supersymmetric. It is possible to fix the VEV of V_5^2 at tree level by using an independent source of supersymmetry breaking that will play the role of the superpotential in the low energy effective theory. This was done in Refs. [12,13] by attaching this superpotential to the branes, i.e. by choosing the flux multiplet with $E_M = \delta_M$, where $\delta_5 = \omega_0 \delta(x^5) + \omega_\pi \delta(x^5 - \pi R)$ and $\delta_\mu = 0$, that obviously satisfies the condition $dE = 0$. We can even generalize the source term by using any fixed closed form as the constant one [9], i.e. $\delta_5 = \omega$. According to the general analysis of Ref. [9] there is nothing special about the orbifold and we can use this particular formalism to implement supersymmetry breaking on the circle. One should of course worry that such term might be breaking general coordinate invariance. In fact it does not since a fixed closed one form is not the same in any frame but it differs by a non-physical gauge transformation. Of course a different issue is the physical origin of the flux multiplet, a point where we are not going to enter here.

In both cases the radion potential is flat at tree-level and we need the use of the Casimir energy to fix it. In Ref. [9] we have thoroughly analyzed the two previous cases. In the case of the tensor multiplet formalism both the SS parameter and the radion VEV have to be determined by the one-

loop effective potential. In that case the two-field minimization leads to a VEV $\omega = 1/2$, independently on the radion VEV. In the case of the linear multiplet formalism ω is fixed at the tree-level by whatever dynamics at the brane or bulk we like, and the one-loop potential provides the radion VEV as a function of the SS parameter. Since the tensor multiplet formalism is more involved and the linear multiplet one allows for arbitrary values of ω, in the following we will concentrate on the latter case.

4. Radion effective potential

We will consider radion stabilization using the Casimir energy. We parametrize the 5D metric in the Einstein frame as [14] $ds^2 = G_{MN}dx^M dx^N \equiv \phi^{-\frac{1}{3}} g_{\mu\nu} dx^\mu dx^\nu + \phi^{\frac{2}{3}} dy^2$. where $y = x^5$ goes from 0 to L. The radion field, whose VEV determines the size of the extra dimension is $\phi^{\frac{1}{3}}$ and the physical radius is given by $R = \langle\phi\rangle^{\frac{1}{3}} L$. The length scale L is unphysical and completely arbitrary. It will drop out once the VEV of the radion is fixed and the effective 4D theory will only depend on R. In order to achieve zero four-dimensional cosmological constant we will introduce bulk cosmological constant g^2 and brane tensions $T_{0,\pi}$ as possible counterterms. This corresponds to AdS_5 supergravity, although the AdS gauge coupling g (as well as the brane tensions) are really one loop counterterms and there is no tree level warping. The four-dimensional effective Lagrangian including the radion one-loop effective potential is $\mathcal{L} = -V + \pi L g^2 \phi^{-\frac{1}{3}} + \frac{1}{2}(T_0 + T_\pi)\phi^{-\frac{2}{3}}$, where V is the Casimir energy.

4.1. *Propagating bulk fields*

By considering N_V vector multiplets and N_h hypermultiplets propagating in the bulk, the Casimir energy is

$$V_{eff} \propto (2 + N_V - N_h)\frac{1}{L^4\phi^2}. \tag{1}$$

Potential (1) is runaway and provides either a repulsive or an attractive force. Of course by adding the counterterms one can create a stable minimum. Unfortunately the required counterterms are not consistent with supersymmetry. The way out is to introduce a mass scale in the theory. This can be done either by introducing a supersymmetric odd mass for some hypermultiplets (that produces an exponential localization of their lightest eigenstate on an orbifold fixed point) and/or by introducing localized kinetic terms [15].

4.2. *Localized effects*

The fields that are strictly localized on the boundary fixed points are four-dimensional fields and as such they cannot influence the Casimir energy. However bulk hyperscalars with brane mass terms and/or localized kinetic terms can influence the bulk Casimir energy while they introduce scales into the theory. In particular the bosonic Lagrangian of a supersymmetric hypermultiplet (φ^i, ψ) with a localized odd parity mass term $M(y) = \eta(y)M$, where $\eta(y)$ is the sign function on S^1 with period πR, and localized kinetic terms can be written as [11]:

$$\begin{aligned}\mathcal{L} = &\left|\mathcal{D}_M\varphi\right|^2 - M^2(y)\left|\varphi\right|^2 + M'(y)(\varphi^\dagger\sigma_3\varphi)\\ &+\frac{2}{M}\left[c_0\delta(\pi) + c_\pi\delta(y-\pi R)\right]\left(\partial_\mu\varphi\right)^2\end{aligned} \tag{2}$$

where the coefficients $c_{0,\pi}$ have been normalized to be dimensionless. The 4D mass spectrum is given by

$$\begin{aligned}\sin^2\omega\pi = &\sin^2(\Omega\pi R)\,\frac{m^2(1-c_0+c_\pi)+c_0c_\pi m^4/M^2}{\Omega^2}\\ &+\frac{m^2(c_0+c_\pi)}{2M\Omega}\sin(2\Omega\pi R)\end{aligned} \tag{3}$$

where $\Omega = \sqrt{m^2 - M^2}$.

The effective potential can be computed using standard methods [16,10]. For the case of N_H hypermultiplets with a common mass M and $c_0 = c_\pi = 0$ it gives the result

$$V_{eff} = M^6 L^2\,\frac{N_H}{8}\int dz\, z^3 \ln\left[1 + \frac{z^2+x^2}{z^2\sinh^2\left(\sqrt{z^2+x^2}\right)}\right] \tag{4}$$

where $x = M\pi R$.

The total one-loop effective potential is a combination of potential (1), corresponding to fields propagating in the bulk, and potential (4) corresponding to bulk fields with localized effects.

5. Radion stabilization

For any value of the SS parameter ω the total potential for the radion field has a global minimum that depends on ω and M. If we now introduce bulk $g^2 > 0$ and tension $T_0 + T_\pi > 0$ counterterms fine-tuned to have zero cosmological constant and consistent with 5D supersymmetry in AdS space, the radion minimum is shifted to a value that depends on the counterterms. On the other hand, as can be seen in Fig. 1, the counterterms introduced

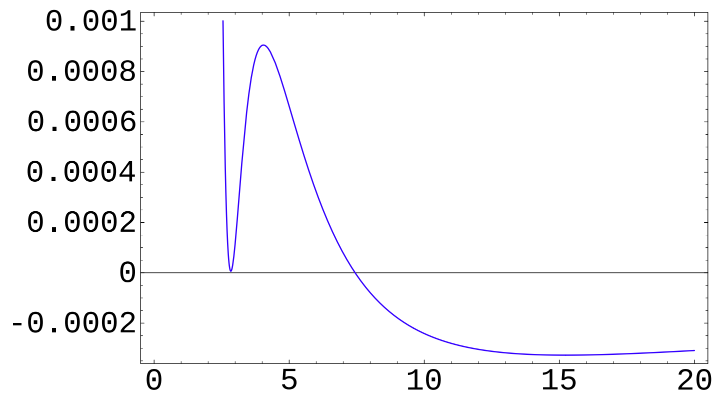

Figure 1. Radion potential for $\omega = 0.25$ as a function of the variable $MR\pi$, for appropriate values of the counterterms.

to cancel the 4D cosmological constant produce an AdS_4 global minimum so that the stability of the Minkowski vacuum becomes a real issue.

The kinetic term for the field ϕ assumes the form $M_4^2(\partial_\mu\phi)^2/\phi^2$ (where M_4 is the 4D Planck scale) and the potential is $M^4 f(\phi)$. In terms of the canonically normalized radion $\varphi = M_4 \ln\phi/\sqrt{3}$ the barrier separating the two vacua, as well as the depth difference ε between them, is of order M^4 while the distance between both minima is of order M_4. Despite the small barrier between the two vacua, in order to tunnel a macroscopic bubble has to be nucleated and the energy cost for it is huge, which leads to exponentially suppressed probability. In fact we can model the potential around the metastable minimum as $V(\varphi) = \lambda(\varphi^2 - M_4^2)^2$, where $\lambda \sim (M/M_4)^4$. The probability in the thin wall approximation [17] is found to be $P \sim \exp[-B]$ where $B \sim M_4^{12}\lambda^2\varepsilon^{-3} \sim (M_4/M)^4 \sim 10^{60}$. We can conclude that the Minkowski vacuum is stable on cosmological times.

6. Radion stabilization and the hierarchy problem

Unlike in those approaches where a warped geometry solves the hierarchy problem, in flat space we must invoke supersymmetry for solving it. Our only concern was to obtain a physical radius $< 1/\text{TeV}$. However this range is technically natural since we are introducing bulk masses in the TeV range. A different (not unrelated) issue is the origin of the weakness of gravitational interactions in the 4D theory and its relation with radion fixing. Here we

have been working in a 5D gravity theory, with a 1/TeV length radius, and therefore the presence of submillimeter dimensions is not consistent with our mechanism for radion stabilization. On the other hand the relation between the Planck scales in the 4D and 5D theories, $M_4^2 = M_5^3 R$, with $R \sim 1/\text{TeV}$ implies that the scale where gravity becomes strong in the 5D theory is much higher than $1/R$. This means that gauge interactions of the 5D theory become non-perturbative at a scale $M_s \ll M_5$ in the multi-TeV range. The theory should then have a cutoff at the scale M_s where a more fundamental theory should be valid. An example of such behaviour is provided by Little String Theories (LST) at the TeV [18,19] where the string coupling $g_s \ll 1$ and M_s and M_5 are related by $M_5^3 = M_s^3/g_s^2$. In other words M_5 does no longer play the role of a fundamental field theoretical cutoff scale. In these theories the weakness of the gravitational interactions is provided by the smallness of the string coupling. Moreover a class of LST has been found [19] where the Yang-Mills coupling is not provided by the string coupling but by the geometry of the compactified space where gauge interactions are localized, e.g. $g_{YM} \sim \ell_s/R$. Since the field theory has a cutoff at M_s the consistency of the whole picture relies on the assumption that there is a wide enough range where the 5D field theory description is valid.

7. Radion phenomenology

In the metastable vacuum the squared mass of the canonically normalized radion field is given by $\sim$ (one-loop factor)$\times M^4/M_4^2$. Since the size of the odd-mass term M may be taken to be of the order of 10 TeV, we conclude that the radion field acquires in the metastable vacuum a mass around $(10^{-3} - 10^{-2})$ eV. This range of masses is interesting for present and future measurements of deviations from the gravitational inverse-square law in the millimeter range [20]. In particular this shows that a positive-signal in table-top gravitational experiments does not necessarily implies the existence of sub-millimeter dimensions.

Finally, we should also be concerned about the backreaction of the Casimir energy and the counterterms on the originally flat 5D gravitational background. A dimensional analysis shows that the effect of the counterterms by themselves would result in a warp factor with a functional dependence on the extra coordinate as $a(\epsilon M y)$, where $\epsilon = \mathcal{O}(M/M_5)^{3/2} \equiv \mathcal{O}(M/M_4) \sim 10^{-15}$ for $M \sim$ TeV. Such a warping is completely negligible. One can also show that the size of the gravitino bulk and brane masses

generated by the counterterms are of the order of the radion mass and thus negligible as compared to the size of supersymmetry breaking contributions.

Acknowledgments

I wish to thank my collaborators G. v. Gersdorff, L. Pilo, D. A. J. Rayner, A. Riotto, for so many discussions we had on this field and the joint work. This work was supported in part by the RTN European Programs HPRN-CT-2000-00148 and HPRN-CT-2000-00152, and by CICYT, Spain, under contracts FPA 2001-1806 and FPA 2002-00748.

References

1. J. Scherk and J. H. Schwarz, *Phys. Lett.* **B82** (1979) 60; *ibidem*, *Nucl. Phys.* **B153** (1979) 61.
2. For a review see: M. Quiros, "New ideas in symmetry breaking", Lectures given at TASI 2002, Boulder, Colorado, 2-28 June 2002, arXiv:hep-ph/0302189.
3. G. von Gersdorff and M. Quiros, *Phys. Rev.* **D65** (2002) 064016 [hep-th/0110132].
4. Y. Hosotani, *Phys. Lett.* **B126**, 309 (1983); *Phys. Lett.* **B129**, 193 (1983).
5. M. Zucker, *Nucl. Phys.* **B570** (2000) 267 [hep-th/9907082]; *JHEP* **0008** (2000) 016 [hep-th/9909144]; *Phys. Rev.* **D64** (2001) 024024 [hep-th/0009083].
6. M. Zucker, *Fortsch. Phys.* **51** (2003) 899.
7. Z. Lalak and R. Matyszkiewicz, *Nucl. Phys.* **B649** (2003) 389 [arXiv:hep-th/0210053]; Z. Lalak and R. Matyszkiewicz, arXiv:hep-th/0310269.
8. J. Bagger and D. Belyaev, *JHEP* **0306** (2003) 013 [arXiv:hep-th/0306063]; J. Bagger and M. Redi, *Phys. Lett.* **B582** (2004) 117 [arXiv:hep-th/0310086].
9. G. von Gersdorff, M. Quiros and A. Riotto, arXiv:hep-th/0310190.
10. G. von Gersdorff, M. Quiros and A. Riotto, *Nucl. Phys.* **B634** (2002) 90 [hep-th/0204041].
11. G. von Gersdorff, L. Pilo, M. Quiros, D. A. J. Rayner and A. Riotto, *Phys. Lett.* **B580** (2004) 93 [arXiv:hep-ph/0305218].
12. J. Bagger, F. Feruglio and F. Zwirner, *JHEP* **0202** (2002) 010 [hep-th/0108010].
13. R. Rattazzi, C. A. Scrucca and A. Strumia, *Nucl. Phys.* **B674** (2003) 171 [arXiv:hep-th/0305184].
14. T. Appelquist and A. Chodos, *Phys. Rev. Lett.* **50** (1983) 141; T. Appelquist and A. Chodos, *Phys. Rev.* **D28** (1983) 772.
15. E. Ponton and E. Poppitz, *JHEP* **0106** (2001) 019 [hep-ph/0105021].
16. A. Delgado, A. Pomarol and M. Quiros, *Phys. Rev.* **D60** (1999) 095008 [hep-ph/9812489]; W. D. Goldberger and I. Z. Rothstein, Phys. Rev. D **68** (2003) 125011 [arXiv:hep-th/0208060].
17. S. R. Coleman and F. De Luccia, Phys. Rev. D **21** (1980) 3305.

18. O. Aharony, *Class. Quant. Grav.* **17** (2000) 929 [hep-th/9911147].
19. I. Antoniadis, S. Dimopoulos and A. Giveon, *JHEP* **0105** (2001) 055 [hep-th/0103033].
20. E. G. Adelberger, B. R. Heckel and A. E. Nelson, arXiv:hep-ph/0307284.